RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College affiliated to University of Calcutta)

SECOND YEAR [2017-20]
B.A./B.Sc. THIRD SEMESTER (July – December) 2018
Mid-Semester Examination, September 2018

Date : 24/09/2018 **MATHEMATICS (Honours)**
Time : 11 am – 1pm Paper: III Full Marks : 50

[Use a separate Answer Book for each group]

GROUP – A

Answer **any three** from questions nos. 1 to 5 : (3 × 5)

1. Prove that every subspace of a finite dimensional vector space over a field F possesses a complement.
2. Examine the nature of intersection of the planes
$$x+y-z=3,\ 5x+2y+z=1 \text{ and } 2x+2y-2z=1$$
3. Find the nature of the following real quadratic form by reducing to normal form:
$$x_1^2+2x_2^2+3x_3^2+2x_1x_2+4x_2x_3+2x_1x_3$$
4. Solve the systems $AX=E_1,\ AX=E_2,\ AX=E_3$ where
$$A=\begin{pmatrix}1&1&0\\0&1&-1\\2&1&2\end{pmatrix},\ E_1=\begin{pmatrix}1\\0\\0\end{pmatrix},\ E_2=\begin{pmatrix}0\\1\\0\end{pmatrix},\ E_3=\begin{pmatrix}0\\0\\1\end{pmatrix}.$$ Hence find A^{-1}.
5. $V=\mathbb{R}^4$ and W be a subspace of V generated by $\{(1,2,3,1),(5,3,1,2)\}$. Find a basis of the quotient space $\frac{V}{W}$. Also verify $\dim\frac{V}{W}=\dim V-\dim W$.

Answer **any two** from question nos. 6 to 8 : (2 × 5)

6. Let $\{f_n\}$ be a sequence of functions on an interval I that converges uniformly on I to a continuous function f. Let $c\in I$ and $\{x_n\}$ be a sequence in I converging to c. Prove that $f_n(x_n)\to f(c)$.
7. For each $i\in\mathbb{N}$ let $\left\{f_n^{(i)}\right\}$ denote a sequence of functions on [0, 1] converging uniformly to a function $f^{(i)}$ (on the same interval). Show that for any $k\in\mathbb{N}$,
$$\max_{1\le i\le k} f_n^{(i)} \to \max_{1\le i\le k} f^{(i)} \text{ uniformly on } (0,1]$$
Is it necessarily true that $\sup_{i\in\mathbb{N}} f_n^{(i)}\to\sup_{i\in\mathbb{N}} f^{(i)}$?
Justify your answer. 3+2
8. State and prove Weirstrass' M-test in the context of uniform convergence of a series of real functions. 1+4

GROUP – B

Answer **any two** from question nos. 9 to 11 : (2 × 5)

9. Let P, Q, R, S be four points in space and L, M, N, T be points dividing the segments $\overline{PQ}, \overline{QR}, \overline{RS}, \overline{SP}$ in the ratios $l:1$, $m:1$, $n:1$ and $t:1$ respectively. If L, M, N, T are coplanar show that $lmnt = 1$.

10. Assuming the plane $4x-3y+7z=0$ be horizontal, find the equations of the line of greatest slope through the point (2, 1, 1) in the plane $2x+y-5z=0$.

11. Find the shortest distance between z-axis and the line: $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z}{n}$.

GROUP – C

Answer **any three** from question nos. 12 to 16: (3 × 5)

12. One end of an elastic string, whose modulus of elasticity is λ and unstretched length is 'a' is fixed to a point on a smooth horizontal table and the other end is tied to a particle of mass 'm' which is lying on the table. The particle is pulled to a distance, where the extension of the string is 'b' and then let go; show that the time of a complete oscillation is $2\left(\pi+\frac{2a}{b}\right)\sqrt{\frac{am}{\lambda}}$.

13. A particle is projected vertically upwards under gravity 'g' with a velocity λV in a medium of resistance k (velocity) per unit mass, V being the terminal velocity. Prove that the greatest height attained by the particle is $\frac{V^2}{g}\left[\lambda-\log(1+\lambda)\right]$ after a time $\frac{V}{g}\log(1+\lambda)$.

14. The range of a rifle bullet is 1200 yards where α is the angle of elevation. Show that if the rifle is fired with the same elevation from a car travelling at 10 m.p.h. towards the target, the range will be increased by $220\sqrt{\tan\alpha}$ feet.

15. A fort is on the edge of a cliff of height h. Find the greatest horizontal distance at which a gun in the fort can hit a ship, if $\sqrt{2gk}$ be the muzzle velocity of the shot. Find also the greatest distance at which a gun in a ship can hit the fort, the muzzle velocity being the same and 'g' is the gravitational force.

16. The velocities of a particle along and perpendicular to the radius vector from a fixed origin are λr and $\mu\theta$ respectively ; find the path and show that the accelerations along and perpendicular to the radius vector are $\lambda^2 r-\mu^2\theta^2/r$ and $\mu\theta\left(\lambda+\frac{\mu}{r}\right)$ respectively.

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